

Frequency Domain Median-like Filter for Periodic and Quasi-Periodic Noise Removal

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ABSTRACT

Removal of periodic and quasi-periodic patterns from photographs is an important problem. There are a lot of sources of this periodic noise, e.g. the resolution of the scanner used to scan the image affects the high frequency noise pattern in the acquired image and can produce moiré patterns. It is also characteristic of gray scale images obtained from single-chip video cameras. Usually periodic and quasi-periodic noise results peaks in image spectrum amplitude. Considering this, processing in the frequency domain is a much better solution than spatial domain operations (blurring for example, which can hide the periodic patterns at the cost of the edge sharpness reduction). A new frequency domain filter for periodic and quasi-periodic noise reduction is introduced in this paper. This filter analyzes the image spectrum amplitude using a local window, checks every spectral coefficient whether it needs the filtering and if so, replaces it with the median taken from the local window. To detect the peaks in the spectrum amplitude, a ratio of the current amplitude value to median value is used. It is shown that this ratio is stable for the non-corrupted spectral coefficients independently of the frequencies they correspond to. So it is invariant to the position of the peaks in the spectrum amplitude. This kind of filtering completely eliminates periodic noise, and shows quite good results on quasi-periodic noise while completely preserves the image boundaries.

Keywords: periodic noise, median filter, spectral filter

1. INTRODUCTION

As it is known, the theory of linear filtering gives optimal solutions, when the corruption can be modeled as a Gaussian process. The mean square error is the accuracy criterion in this case. These assumptions are reasonable in many practical tasks and the corresponding applications. On the other hand the errors in digital systems are often caused by bit changes, and the resulting distribution is far from the Gaussian one. Also in many cases, where visual quality is the ultimate criterion, the mean square error is not the best objective quality criterion: an image with a larger mean square error can be visually accepted better than one with a smaller mean square error. This is the case, where the nonlinear filtering methods are the most useful¹.

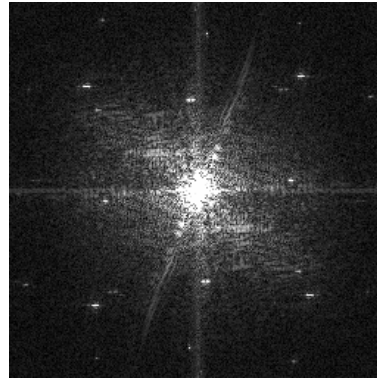
We will consider a problem of the periodic and quasi-periodic noise filtering in this paper. Removal of the periodic and quasi-periodic corrupting patterns from the images is an important problem. The issues of this noise are different. For example, the resolution of the scanner used to input the image usually affects the high frequency noise pattern in the acquired image and can often produce the additional moiré patterns. This artifact is also characteristic of gray scale images obtained from single-chip video cameras.

A usual approach for the quasi-periodic noise filtering is the frequency domain processing. It is a much better solution than any kind of spatial domain blurring, which often is unable to reduce a periodic structure, but at the same time it reduces the edge sharpness significantly. Usually any periodic and quasi-periodic noise leads to the appearance of peaks in the image spectrum amplitude (See Fig.1). This is true for the Fourier, Cosine and Walsh spectra, for example. In the case of periodic noise, these peaks are even visible to an eye, while the peaks corresponding to the quasi-periodic noise usually are hidden and sometimes it is difficult to detect them without some kind of numeric analysis.

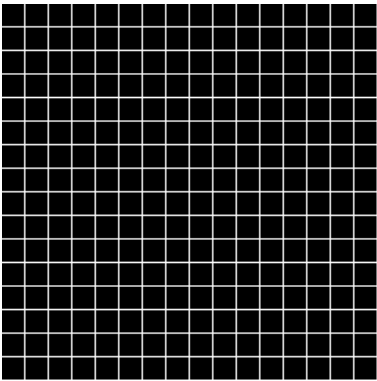
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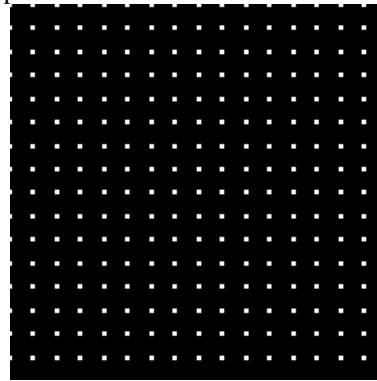
(a) The input image corrupted by the quasi-periodic noise



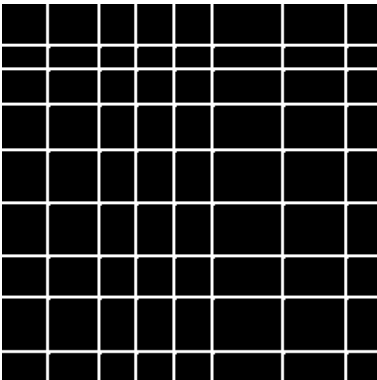
(b) The Fourier spectrum amplitude of the image (a): the bright white spots and “scratches” are the peaks implied by the quasi-periodic noise. Not strongly periodic character of the noise leads to the quasi-random distribution of the peaks, moreover, some of the peaks are small and invisible



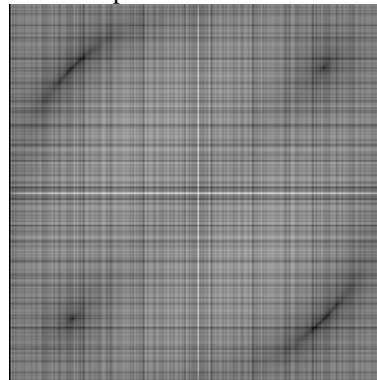
(c) The pure periodic structure: grid



(d) The Fourier spectrum amplitude of the image (c): the white pixels are the peaks implied by the noise. A strongly periodic character of the grid leads to the strongly periodic structure of the peaks



(e) The quasi-periodic structure



(f) The Fourier spectrum amplitude of the image (g): the white pixels are the peaks implied by the noise. Not periodic character of the noise leads to the small peaks in the spectrum amplitude. The peaks could not be detected “manually”

Fig.1. Fourier spectrum amplitude of the images with the periodic noise (images to the left, amplitudes to the right)

A good idea is to find some algorithm for the peaks detection and elimination. Until now we had seen only two solutions: the Wiener filter² and the spectrum domain peak elimination using a mask created by the thresholding of the spectrum amplitude^{3,4}. The Wiener filter is good in general, but to apply this filter, a proper and precise noise model must be built, which is a complicate task. Furthermore, the Wiener filter is computationally slow. The second approach, which is reduced to the spectrum amplitude thresholding, can't be considered as an optimal solution. This approach is suitable only for the

truly periodic noise, which implies high peaks in the image spectrum amplitude, while quasi-periodic noise doesn't give the strong peaks that can be detected using the thresholding.

This way the target of our work is to develop a different method. We are trying to present here more or less general solution, which will cover both periodic and quasi-periodic noise filtering. The proposed algorithm consists of the periodic and quasi-periodic noise frequency domain automatic detection and further elimination. A very strong criterion for the automatic detection even of the small peaks in the spectrum amplitude will be developed. After the peaks' detection the median filter will be used for their elimination. To present a way, in which we came to the results, the preliminary research will be also presented.

The preliminary work was concentrated on the development of the masked mean frequency domain filter. This filter is based on the comparison of the analyzed spectral coefficient's amplitude to the mean value of the selected pixels within a local window around the analyzed pixel. This approach to the peaks' detection shows very good results for the periodic noise. At the same time it is not so good for the quasi-periodic noise. To find a solution, which will be good for the quasi-periodic noise, we changed the criterion for the peaks detection: the comparison of the spectral coefficient's amplitude to the mean value of the selected pixels within a local window has been replaced by the comparison to the median taken from this window.

2. MASKED SPECTRAL MEAN FILTER

The idea behind this filter is to analyze the image spectrum amplitude using a scanning window of a size $N \times N$. To choose the elements within the window for the comparison, a mask is defined. This mask can be defined as a binary matrix of a dimension $N \times N$. The choice of the elements is organized in the following way. A local window has to be covered by the mask. If the element of the mask corresponding to the spectrum amplitude value in the local window equals to 1 then this value is included in the local mean calculation. The central element of the window, which is the amplitude of the spectral coefficient to be analyzed and filtered (the central coefficient within the window), must be excluded from the local mean calculation. It means that the corresponding element of the mask must be equal to zero. I.e.:

$$M = \begin{pmatrix} m_{00} & \dots & m_{0,N-1} \\ \cdot & & \cdot \\ \cdot & 0 & \cdot \\ \cdot & & \cdot \\ m_{N-1,0} & \dots & m_{N-1,N-1} \end{pmatrix}; \quad m_{ij} = \{0,1\}, m_{\frac{N-1}{2}, \frac{N-1}{2}} = 0; \quad (1)$$

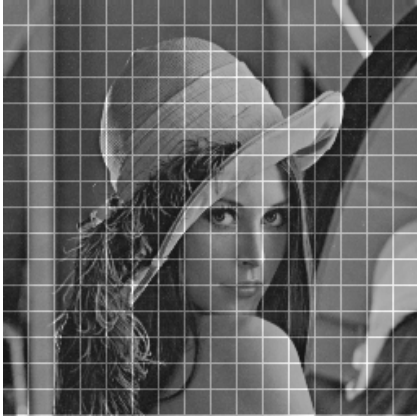
$$S_{kl} = \frac{\sum_{i=-\frac{N}{2}}^{\frac{N}{2}} \sum_{j=-\frac{N}{2}}^{\frac{N}{2}} m_{ij} |X_{k+i, l+j}|}{T}$$

where M is the mask, X is a spectrum amplitude, T is a number of nonzero elements in M , S_{kl} is a mean in a local $N \times N$ window around the $(k, l)^{\text{th}}$ spectral coefficient. The filtering criterion is the following:

$$\frac{X_{kl}}{S_{kl}} > \Theta, \quad (2)$$

where Θ is some threshold. The value of Θ may be evaluated from the analysis of the spectra corresponding to the clean, not noisy images. Simple estimation shows that the following condition holds for the spectra without the peaks implied by the periodic noise: $\frac{X_{kl}}{S_{kl}} < 5$ (for the window of 3×3 dimensions). This is true for the Fourier, Cosine and Walsh spectra. It

means that it is a good idea to choose $\Theta > 4$ in (2). If the condition (2) holds, the value X_{kl} of the spectrum amplitude is considered as a "peak". This peak has to be eliminated. To implement this elimination, the following procedure is proposed:



(a) The input image corrupted by the rectangular periodic noise (grid)



(b) Image (a) filtered by (2)-(3): the image is completely cleaned from grid.



(c) The input image corrupted by the diagonal periodic noise



(d) Image (c) filtered by (2)-(3), the stripes are removed



(e) The input image corrupted by the dark and bright stripes



(f) Image (e) filtered by (2)-(3), the stripes are reduced and became almost invisible

Fig.2. Examples of the Masked Spectral Mean Filter application

$$\hat{X}_{kl} = \frac{X_{kl}}{\delta}, \quad (3)$$

where δ is a normalizing divider. A choice of δ has to lead to the elimination of the peak, but not to the situation, when $\frac{X_{kl}}{\delta} \rightarrow 0$. Otherwise we will replace one peak by another one.

It will be clear from the results shown below that this filter gives the excellent results for the periodic noise, which makes very sharp peaks in the image spectrum amplitude, while for the quasi-periodic noise it is impossible to achieve the good results. Sometimes this filter is also a bit sophisticated from the point of view of parameters selection, especially the selection of δ in (3). Figure 2 shows the filtering results. The Fourier transform has been used in all examples. The following parameters have been used in (1) - (3):

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \Theta = 5, \quad \delta = 50.$$

3. SPECTRAL MEDIAN FILTER

This filter, just as the previous one, operates on the image spectrum. It is closely related to the spatial domain median filter¹. It uses the same technique, the only difference is that it operates on the image spectrum amplitude and checks whether the spectral coefficient requires correction before doing any filtering.

The spectrum amplitude is being filtered using a moving window of $m \times n$ dimensions. Every spectral coefficient's amplitude is being checked to the value of the local window's median and decided upon whether to filter it or not. To be more precise, the current coefficient's amplitude to a local median ratio is what we need. Every peak in the spectrum amplitude corresponding to the quasi-periodic noise is strongly higher than the neighboring amplitude values only in the rare cases. This case is presented in Fig. 5, where a pure periodic noise and its Fourier spectrum amplitude are shown. The sharp peaks are clearly visible.

But usually peak looks like a hill surrounded by the smaller hills. In this case the median filtering with a small window (3x3) will have almost no effect. This is the property of the median filter applied to the signal with the mentioned property¹. Thus it is a good idea to choose a larger window, beginning from 5x5.

The filter itself can be represented by the following formula:

$$\hat{X}_{ij} = \begin{cases} \underset{m \times n}{MED}(X_{ij}), & \text{if } \frac{X_{ij}}{\underset{m \times n}{MED}(X_{ij})} \geq \Theta \quad \text{and not } j = i = 0 \\ X_{ij}, & \text{otherwise} \end{cases} \quad (4)$$

where X is the image spectrum amplitude, \hat{X} is the resulting amplitude after the filtering, (i,j) – coordinates of the coefficient being filtered, $\underset{m \times n}{MED}(X_{ij})$ – median in the local $m \times n$ window around the coefficient being filtered, Θ is the threshold. Evidently, according to the eq. (4) the filter is not applied to the correction of the spectral coefficient X_{00} (because it represents only the mean brightness of the image, and it is a peak that has to be left alone).

As it was already mentioned, a ratio $\frac{X_{ij}}{\underset{m \times n}{MED}(X_{ij})}$ for the image without any periodic structures is quite stable. For

example, for a 5x5 window and the Fourier spectrum amplitude it is usually less than 3, for a 11x11 window it's about 6, but there can be exceptions. So it is always a wise decision to try different thresholds. A window choice is heavily bound to the size of the peak. It means that to eliminate the peak, the window must be larger than a "hill" area around the peak. For example, let us consider the image in Fig. 5a. Its Fourier spectrum amplitude is shown in Fig. 5c. One can notice that the contaminated points are positioned on the spectrum's diagonal and every point represents about 1-4 spectral coefficients, thus it is a good idea to use a 5x5 window. In this case every peak will be completely covered by the window. At the same time for the spectrum amplitude (Fig. 6c) of the image presented in Fig. 6a a larger window is needed because the corresponding

peaks are “fuzzy”. This follows from the character of the noise. To take a closer look on the application of the spectral median filter and its potential opportunities, consider examples in Fig.3-6.

4. CONCLUSIONS AND FUTURE WORK

A problem of the periodic and quasi-periodic noise filtering has been considered in this paper. This kind of noise implies the appearance of the peaks in image spectrum amplitude. So the noise reduction is reduced to elimination of these peaks. The existing techniques of Wiener filtering and elimination of the peaks using binary mask are not good enough. The Wiener filter is powerful and effective, but it is too complicate from the computing point of view. The masking of the spectrum amplitude strongly depends on choice of the threshold parameters and it is effective just for the pure periodic noise with a relatively long period.

So our target was to develop a new effective filter with a semi-automatic detector of the peaks in the spectrum amplitude. Another requirement, which was important for us, is a computing simplicity, for example in comparison to the Wiener filter. It was succeeded to achieve the good results, which satisfy the mentioned requirements.

The spectral mean filter has been developed on the preliminary stage of the work. This filter detects the peaks in the spectrum amplitude comparing a ratio between the current amplitude value and the mean value of the selected pixels within a 3x3 window to the threshold parameter. The detected peaks are replaced by the corresponding means. This approach has shown the good results for the pure periodic noise removal.

To reduce the quasi-periodic noise, the spectral median filter has been developed. It is quite fast from the computing point of view. It allows achieving very good results for the periodic and quasi-periodic noise filtering. The filter removes peaks from the image spectrum, replacing them by the median taken from a local window around the analyzed spectrum amplitude value.

The further work will be directed to the improvement of the developed solution. Since we use the spectral transforms whose coefficients are ordered according to the frequencies, there is still one problem. It is known that median filter preserves the edges¹ of the filtered image. It also preserves the “edges” of the filtered image spectrum amplitude. It means that the “hill” in the spectrum corresponding to the image corrupted by the complicate quasi-periodic structure that has a wide base will be preserved in general after the filtering. Only disturbances in this hill will be eliminated using the proposed technique, not the hill itself. At the same time our intension is to remove this hill. The coefficients of the image spectra (i.e. Fourier, Cosine, Walsh with Walsh ordering) descend with the growth of frequency, in general. Thus any different behavior of the spectrum (which usually is represented by the large hills) must be eliminated and put into some allowed limits.

This is the main subject of the future work that is to be done.



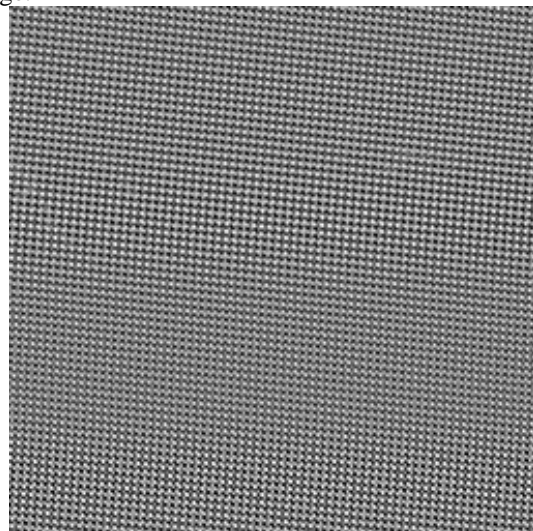
(a) The input image corrupted by the quasi-periodic structure



(b) Image (a), filtered by (4) with a 5x5 window and $\Theta = 3$. Most of the periodic structure is removed, but one can notice a new structure - dark fuzzy stripe over the image.



(c) Image (a), filtered by (4) with a 11x11 window and $\Theta = 6$. A 11x11 window gives much better results than a 5x5 window. We tried different windows and thresholds and we had to seek the compromise between the removal of the input noise and adding the new one. In the case of 11x11 window, no new periodic structure appears



(d) The enhanced difference between (a) and (c)

Fig. 3. Testing image “Clock” corrupted by the quasi-periodic structure: filtering using the spectral median filter



(a) The input image corrupted by the rectangular periodic noise



(b) Image (a), filtered by (4) with a 5×5 window and $\Theta = 3$. The truly periodic noise is completely removed leaving no trail.



(c) The input image corrupted by the quasi-periodic noise



(d) Image (c), filtered by (4) with a 5×5 window and $\Theta = 3$. The noise is almost gone again.

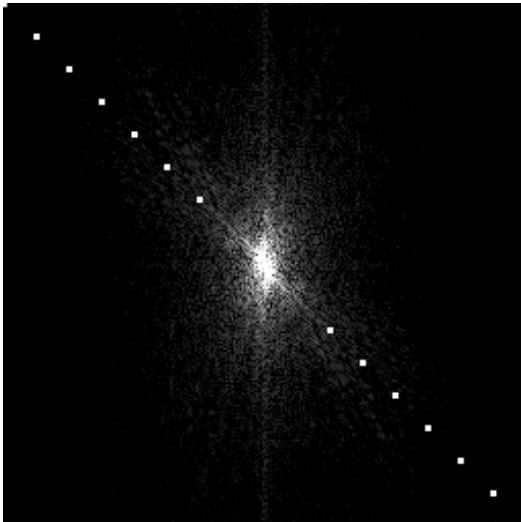
Fig.4. Testing image “Lena” corrupted by the rectangular periodic and quasi-periodic structures:
filtering using the spectral median filter



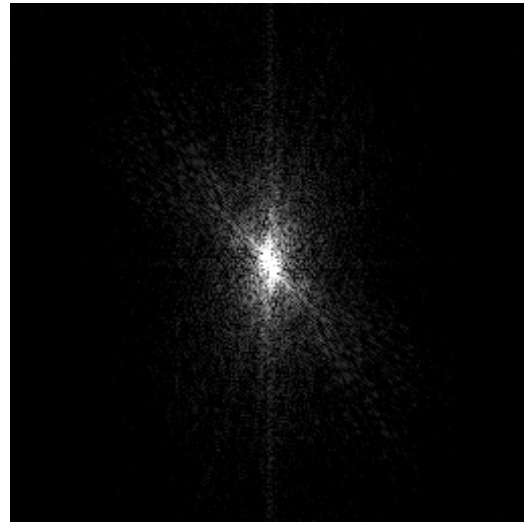
(a) The input image corrupted by the diagonal periodic noise



(b) Image (a), filtered by (4) with a 5x5 window and $\Theta = 3$. The diagonal periodic noise is completely removed.



(c) The Fourier Spectrum amplitude of the image (a)



(d) The Fourier Spectrum amplitude of the image (b) after the filtering

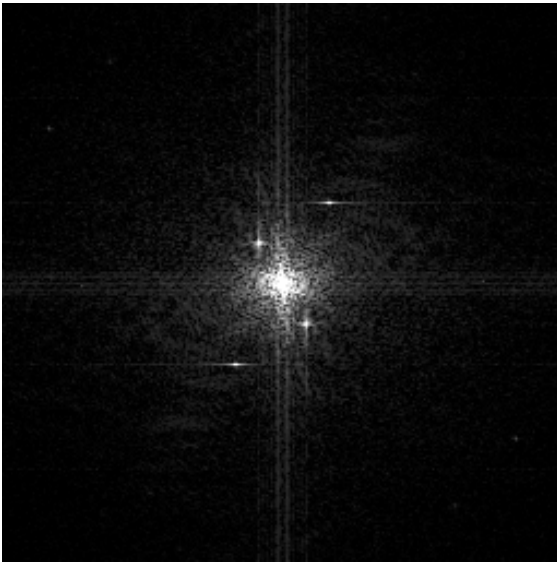
Fig.5. Testing image “Lena” corrupted by the diagonal periodic structure:
filtering using the spectral median filter



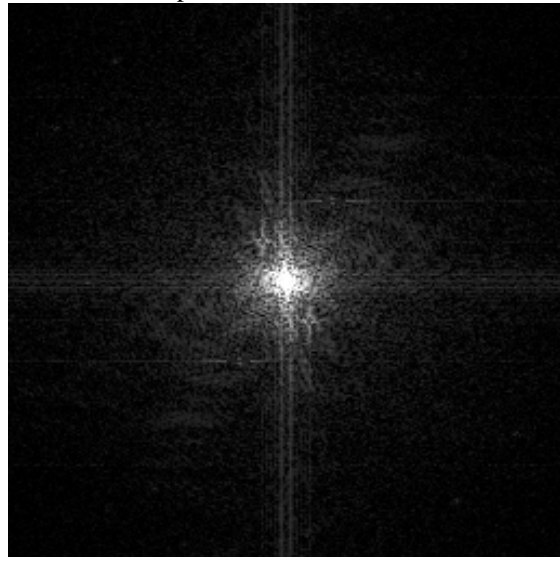
(a) The input image corrupted by the quasi-periodic noise



(b) Image (a), filtered by (4) with a 11x11 window and $\Theta = 6$. Most of the periodic structure is removed.



(c) The Fourier Spectrum amplitude of the image (a)



(d) The Fourier Spectrum amplitude of the image (b) after the filtering

Fig. 6. Testing image “Clown” corrupted by the quasi-periodic noise: filtering using the spectral median filter

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